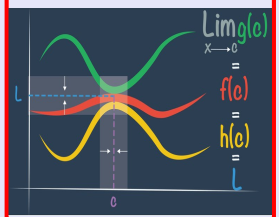
Calculus I

Lecture 53



Feb 19-8:47 AM

Find $\int x^2 \cos x^3 \, dx$

Let $u = x^3$

$du = 3x^2 dx$

$\frac{du}{3} = x^2 dx$

$= \int \cos u \, \frac{du}{3}$

$= \frac{1}{3} \int \cos u \, du = \frac{1}{3} \sin u + C$

$= \boxed{\frac{1}{3} \sin x^3 + C}$

To verify

$\frac{d}{dx}\left[\frac{1}{3} \sin x^3 + C\right] \stackrel{?}{=} x^2 \cos x^3$

May 21-8:45 AM

Find $\int \frac{\sin\frac{\pi}{x}}{x^2}\,dx$

Let $u = \frac{\pi}{x}$

$du = \frac{-\pi}{x^2}\,dx$

$\frac{du}{-\pi} = \frac{1}{x^2}\,dx$

$= \int \sin u \,\frac{du}{-\pi}$

$= \frac{-1}{\pi}\int \sin u \,du = \frac{-1}{\pi}\cdot -\cos u + C$

$= \boxed{\frac{1}{\pi}\cos\frac{\pi}{x} + C}$

To verify

$\frac{d}{dx}\left[\frac{1}{\pi}\cos\frac{\pi}{x} + C\right] \stackrel{?}{=} \frac{\sin\frac{\pi}{x}}{x^2}$

May 21-8:49 AM

Evaluate $\int_{-1}^{2} x^2\sqrt{2+x}\,dx$

Let
$u = 2+x \rightarrow x = u-2$
$du = dx$

$= \int_1^4 (u-2)^2\sqrt{u}\,du$

$x=-1 \rightarrow u=1$
$x=2 \rightarrow u=4$

$= \int_1^4 (u^2-4u+4)\sqrt{u}\,du = \int_1^4 (\underbrace{u^2\sqrt{u}}_{u^{5/2}} - 4\underbrace{u\sqrt{u}}_{u^{3/2}} + 4\underbrace{\sqrt{u}}_{u^{1/2}})\,du$

$= \left[\frac{u^{7/2}}{7/2} - 4\cdot\frac{u^{5/2}}{5/2} + 4\cdot\frac{u^{3/2}}{3/2}\right]_1^4$

$= \left(\frac{2}{7}u^3\sqrt{u} - \frac{8}{5}u^2\sqrt{u} + \frac{8}{3}u\sqrt{u}\right)\Big|_1^4 = \boxed{}$ ✓

Try $u = \sqrt{2+x}$
$u^2 = 2+x \rightarrow x = u^2-2$
$2u\,du = dx$

$x=-1 \rightarrow u=1$
$x=2 \rightarrow u=2$

$\int_{-1}^{2} x^2\sqrt{2+x}\,dx = \int_1^2 (u^2-2)^2\,u\,2u\,du = 2\int_1^2 (u^4-4u^2+4)u^2\,du$

$= \boxed{}$ ✓

May 21-8:54 AM

Find the interval where $f(x)=\int_0^x \frac{t^2}{t^2+t+2}\,dt$

is concave upward.

$$\underbrace{\text{concave upward}}_{f''(x)>0}$$

$$f'(x)=\frac{x^2}{x^2+x+2}\cdot 1-\frac{0^2}{0^2+0+2}\cdot 0$$

$$f'(x)=\frac{x^2}{x^2+x+2}$$

$$f''(x)=\frac{2x(x^2+x+2)-x^2(2x+1)}{(x^2+x+2)^2}$$

$$f''(x)=\frac{x^2+4x}{(x^2+x+2)^2}$$

$f''(x)=0$ $\quad x^2+4x=0$

$x=0\ ,\ x=-4$

$f''(x)$ und. $\quad x^2+x+2=0$

$$x=\frac{-1\pm\sqrt{1-8}}{2}$$

No real Solution

Use sign chart

x		-4		0	
$f''(x)$	$+$		$-$		$+$
$f(x)$	C.U.		C.D.		C.U.

May 21-9:07 AM

Find c in $[0,1]$ such that $f(c)=f_{ave}$

for $f(x)=x^2$ on $[0,1]$.

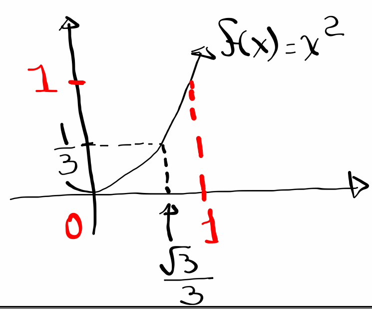
$f(c)=c^2$

$$f_{ave}=\frac{1}{1-0}\int_0^1 x^2\,dx$$

$$=\left.\frac{x^3}{3}\right|_0^1=\frac{1}{3}$$

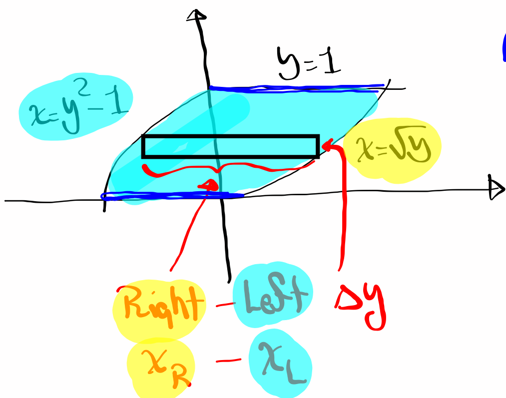
$f(c)=f_{ave}$

$c^2=\frac{1}{3}\ \rightarrow\ c=\sqrt{\frac{1}{3}}\qquad c=\frac{\sqrt{3}}{3}$



May 21-9:15 AM

Find the area of the shaded region below



$$A = \int_0^1 \left[\sqrt{y} - (y^2-1)\right] dy$$

$$= \int_0^1 (\sqrt{y} - y^2 + 1)\, dy$$

$$= \boxed{}$$

May 21-9:20 AM

Find the volume if the region enclosed by

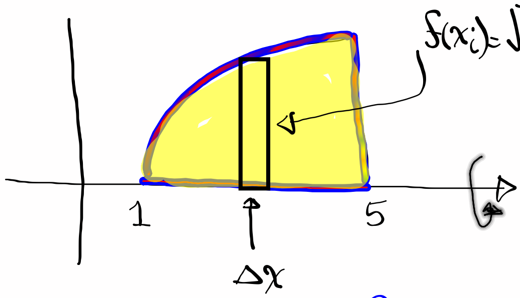
$\boxed{y = \sqrt{x-1}}$, $\boxed{y = 0}$, and $\boxed{x = 5}$ is rotated

about x-axis.

✓1) Ref. Rect. ⊥ A.O.R.

✓2) Region is totally attached to A.O.R.

Disk Method



$$V = \int_1^5 \pi \left[\sqrt{x-1}\right]^2 dx = \pi \int_1^5 (x-1)\, dx = \boxed{}$$

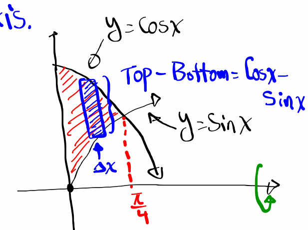
May 21-9:27 AM

Rotate the region bounded by $y = \sin x$, $y = \cos x$ on $[0, \frac{\pi}{4}]$ about x-axis.

Find the Volume.

Ref. Rect. $\perp$ A.O.R.

Region is not totally attached to A.O.R.



we cannot use disk → we should use washer Method.

Washer Method

$$V = \int_a^b \pi \left[Top^2 - Bottom^2 \right] dx$$

$$= \int_0^{\pi/4} \pi \left[\underbrace{\cos^2 x - \sin^2 x}_{\cos 2x} \right] dx = \pi \int_0^{\pi/4} \cos 2x \, dx$$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

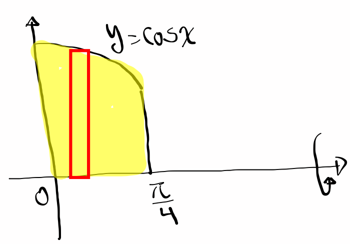
$x = 0 \to u = 0$

$x = \pi/4 \to u = \frac{\pi}{2}$

$$= \pi \int_0^{\pi/2} \cos u \cdot \frac{du}{2}$$

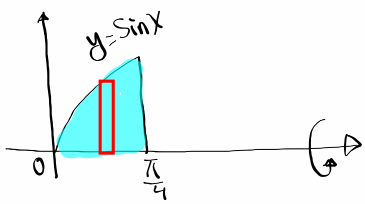
$$= \frac{\pi}{2} \cdot \sin u \Big|_0^{\pi/2} = \frac{\pi}{2}(1 - 0) = \boxed{\frac{\pi}{2}}$$

May 20-9:43 AM

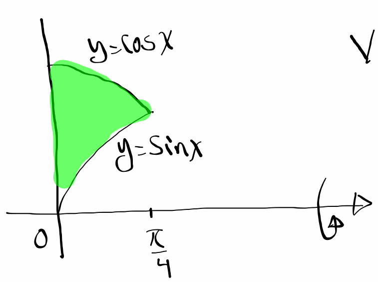


Disk Method

$$V_1 = \int_0^{\frac{\pi}{4}} \pi \cos^2 x \, dx$$



$$V_2 = \int_0^{\frac{\pi}{4}} \pi \sin^2 x \, dx$$



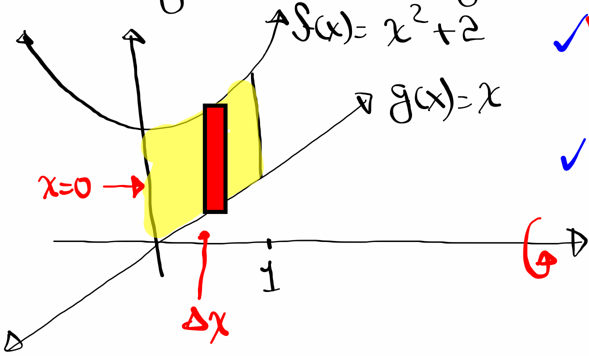
$$V = V_1 - V_2$$

$$= \int_0^{\pi/4} \pi \left[\cos^2 x - \sin^2 x \right] dx$$

Washer Method

May 21-9:34 AM

Find the volume of the solid by rotating the region below by x-axis.



✓ 1) Ref. Rect. ⊥ A.O.R.

✓ 2) Region is not totally attached to A.O.R

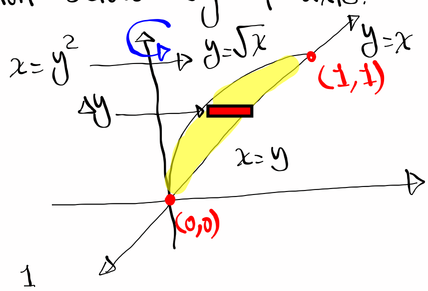
Washer Method

$\pi[\text{Top}^2-\text{Bottom}^2]\Delta x$

$$V=\int_0^1 \pi\left[(x^2+2)^2-(x)^2\right]dx = \boxed{}$$

May 21-9:40 AM

Find the volume by rotating the shaded region below by Y-axis.



1) Ref. Rect. ⊥ AOR

2) Region is not totally attached to AOR.

Washer Method

$$V=\int_0^1 \pi[\text{Right}^2-\text{Left}^2]dy=\int_0^1 \pi[y^2-y^4]dy$$

$$=\pi\left[\frac{y^3}{3}-\frac{y^5}{5}\right]\Bigg|_0^1=\pi\left[\frac{1}{3}-\frac{1}{5}-0\right]$$

$$=\boxed{\frac{2\pi}{15}}$$

May 21-9:47 AM